**MATHEMATICS METHODS**

**MAWA Semester 1 (Unit 3) Examination 2018**

**Calculator-assumed**

# Marking Key

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The release date for this exam and marking scheme is

* **the end of week 8 of term 2, 2018**

**Section Two: Calculator-assumed (100 Marks)**

**Question 9 (5 marks)**

|  |  |
| --- | --- |
| Solution | |
| ➀  ➁ | |
| Mathematical behaviours | Marks |
| * uses (0,- 6) to determine *d* * differentiates and uses  to obtain *c* * states and states relationship between *a* and *b* * uses (1,-8) to determine relationship between *a* and *b* * solves simultaneous equations to determine *a* and *b* | 1  1  1  1  1 |

**Question 10(a) (4 marks)**

|  |  |
| --- | --- |
| Solution | |
| Let denote the number of questions that Fiona answers correctly, assuming that she is guessing. Then is probability that Fiona passes.  Similarly, if denotes the number of questions that Gary answers correctly, assuming that he is guessing, then is probability that Gary passes.  Now and  and  Since the probability that Gary passes via guessing is less than the probability that Fiona passes via guessing, we can say that Gary is luckier. | |
| Mathematical behaviours | Marks |
| * recognizes the binomial probabilities * evaluates probabilities * justifies who is luckier | 1  1+1  1 |

**Question 10 (b) (i) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| Let denote the number of light bulbs that fail in a random sample of 100. Then , if the manufacturer is correct.  Then | |
| Mathematical behaviours | Marks |
| * recognizes binomial probability with correct parameters * states probability | 1  1 |

**Question 10 (b) (ii) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| Because the probability that such a large number of bulbs fail if the manufacturer’s claim is correct, is very, very small, there is strong reason to doubt the validity of the claim. | |
| Mathematical behaviours | Marks |
| * correct conclusion * justifies reasoning | 1  1 |

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * recognises and states exponential growth formula for *I* * uses relationship * states solution | 1  1  1 |

**Question 11 (3 marks)**

**Question 12 (a) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| For each die this occurs with probability  Since the dice are independent, the probability that this occurs both dice is | |
| Mathematical behaviours | Marks |
| * observes that both numbers must be at most *n* * uses independence to justify multiplicative formula | 1  1 |

**Question 12 (b) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| From part (a)  So Vanessa’s expected winnings from each $1 she bets is  So her expected return from 100 $1 bets is a loss of | |
| Mathematical behaviours | Marks |
| * correct expected value for a $1 bet. * correct final answer | 1  1 |

**Question 12 (c) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| |  |  | | --- | --- | |  |  | | 1 | 1/36 | | 2 | 4/36 – 1/36 = 3/36 | | 3 | 9/36 – 4/36 = 5/36 | | 4 | 16/36 – 9/36 = 7/36 | | 5 | 25/36 – 16/36 = 9/36 | | 6 | 36/36 – 25/36 = 11/36 | | |
| Mathematical behaviours | Marks |
| * uses subtraction to obtain individual probabilities from the cumulative   ones in part (a)   * correct answers in all 5 outstanding cases | 1  1 |

**Question 12 (d) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| |  |  |  | | --- | --- | --- | |  |  |  | | 1 | 1/36 | 1/36 | | 2 | 3/36 | 6/36 | | 3 | 5/36 | 15/36 | | 4 | 7/36 | 28/36 | | 5 | 9/36 | 45/36 | | 6 | 11/36 | 66/36 |   Directly from calculator, or via: | |
| Mathematical behaviours | Marks |
| * writes a calculation for expected value * determines expected value | 1  1 |

**Question 12 (e) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| |  |  |  | | --- | --- | --- | |  |  |  | | 1 | 1/36 | 1/36 | | 2 | 3/36 | 12/36 | | 3 | 5/36 | 45/36 | | 4 | 7/36 | 112/36 | | 5 | 9/36 | 225/36 | | 6 | 11/36 | 396/36 |   Directly from calculator, or via | |
| Mathematical behaviours | Marks |
| * calculates  correctly * calculates variance correctly | 1  1 |

**Question 12 (f) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| If a large number of dice are thrown, *m,* say, then    So is almost certainly equal to 6  So | |
| Mathematical behaviours | Marks |
| * correct answer for * correct answer for | 1  1  1 |

**Question 13 (a) (4 marks)**

|  |  |
| --- | --- |
| Solution | |
| Let represent the yield from the trees  Let represent the number of extra trees planted    Macintosh orchard should plant an additional 20 trees  for optimal yield. | |
| Mathematical behaviours | Marks |
| * clearly identifies variables used in equation * correct ‘yield’ equation * differentiates and solves * justifies that maximum is found and states solution | 1  1  1  1 |

**Question 13 (b) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
| Current Yield: 27 300 oranges  Optimal Yield: 28 900 oranges    Therefore there would be a 5.9% increase in yield after planting 10 additional trees. | |
| Mathematical behaviours | Marks |
| * states correct answer | 1 |

**Question 14 (a) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| upper limit    lower limit    represents the area under the curve from *x* = 0 to *x*= 6, bounded by the *x* axis. The exact area will lie between the upper limit and lower limit. | |
| Mathematical behaviours | Marks |
| * shows a calculation to determine an upper limit * shows a calculation to determine a lower limit * explains the limits in terms of area | 1  1  1 |

**Question 14 (b) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
| Using more rectangles would enable the rectangles to more closely approximate the shape of the function. Hence the error involved in approximating  is less and the interval obtained will decrease. | |
| Mathematical behaviours | Marks |
| * explains why the interval will decrease | 1 |

**Question 14 (c) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * states correct answer to 4 significant figures | 1 |

**Question 15 (6 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| For   * point of inflection is on *y* axis * maxima/minima align with roots of * correct shape   For *f(x)*   * stationary points align with *x*-intercepts from *f ‘(x)* * points of inflection align with the roots of * correct shape | 1  1  1  1  1  1 |

**Question 16 (a) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| The rate of change of displacement with  respect to time at 5 seconds is 12 m/s. | |
| Mathematical behaviours | Marks |
| * determines *s’(t)* * determines the rate of change with units | 1  1 |

**Question 16 (b) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * solves * demonstrates * states interval with correct symbols | 1  1  1 |

**Question 17 (a) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| |  |  | | --- | --- | |  |  | |  |  | |  |  | |  |  |   The required probabilities are the ratios of the numbers of favourable choices to the number of all possible choices.  The number of all possible choices is | |
| Mathematical behaviours | Marks |
| * uses combinations to determine numerators * uses combinations to determine denominators * evaluates all probabilities | 1  1  1 |

**Question 17 (b) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| So  In summary, the expected value of is 1.2 and the variance is 0.36 | |
| Mathematical behaviours | Marks |
| * evaluates * evaluates * evaluates | 1  1  1 |

**Question 17 (c) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| has a binomial distribution because it represents the sum of two independent trials (choosing mugs) with the same probability of ‘success’ in each trial  does not have a binomial distribution because the trials are not independent, i.e. the outcome of the first trial affects the probabilities in the second trial | |
| Mathematical behaviours | Marks |
| * independence of trials noted (for * unchanged probabilities noted (for ) * probabilities for the second choice affected by the outcome of the first   choice (for | 1  1  1 |

**Question 17 (d) (5 marks)**

|  |  |
| --- | --- |
| Solution | |
| |  |  | | --- | --- | |  |  | |  |  | |  |  | |  |  |   The correct answer is the expected value of  if and only if the mug chosen is unchipped and exactly 1 of the previous chosen mugs is unchipped. So  So  So Caitlin can expect to choose, on average, 3 mugs. | |
| Mathematical behaviours | Marks |
| * recognizes as the correct answer * evaluates individual probabilities * evaluates | 1  1+1+1  1 |

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * determines | 1 |

**Question 18 (a) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
|  |  |
| Mathematical behaviours | Marks |
| * recognizes and applies the fundamental theorem * identifies three stationary points | 1  1 |

**Question 18 (b) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
|  |  |
| Mathematical behaviours | Marks |
| * identifies one interval for which *F* is increasing * states, with correct symbols, both intervals for which *F* is increasing | 1  1 |

**Question 18 (c) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| Where points of inflection occur Hence | |
| Mathematical behaviours | Marks |
| * states * states the approximate *x* value () of both stationary points | 1  1 |

**Question 18 (d) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * determines one *x* co-ordinate of intersections * determines all *x* co-ordinates of intersections * states appropriate integral to determine area * determines exact area | 1  1  1  1 |

**Question 19 (4 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * anti-differentiates correctly * uses initial conditions to establish *c* = 0 | 1  1 |

**Question 20 (a) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| If the particle changes direction,  Hence it does not change direction | |
| Mathematical behaviours | Marks |
| * equates * solves equation and states that particle does not change direction | 1  1 |

**Question 20 (b) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| Hence average speed = 15*ms*-1 | |
| Mathematical behaviours | Marks |
| * states integral required to determine total distance travelled * determines average speed | 1  1 |

**Question 20 (c) (2 marks)**

**Question 20 (d) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| Hence the particle is moving with a  positive velocity and is gaining speed | |
| Marking key/mathematical behaviours | Marks |
| * evaluates at least one of *v*(*2*) and * states the particle is moving with a positive velocity/to the right and is speeding up | 1  1 |

**Question 21 (a) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * determines expression | 1 |

**Question 21 (b) (4 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * differentiates expression * uses and * obtains expression for * determines approximate increase in metal required including unit | 1  1  1  1 |

**Question 22 (a) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| In the northern hemisphere highest temperatures occur in the middle of the year, whereas in the southern hemisphere highest temperatures occur at the beginning and end of the year. Since the data show high temperatures in the middle of the year, the city is more likely to be in the northern hemisphere. | |
| Mathematical behaviours | Marks |
| * states more likely hemisphere * valid reasoning | 1  1 |

**Question 22 (b) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| The average maximum temperature values are    Mean =  So the estimated average maximum temperature  is | |
| Mathematical behaviours | Marks |
| * states an appropriate calculation to determine the mean * determines the mean | 1  1 |

**Question 22 (c) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| The estimated standard deviation of the temperatures is | |
| Mathematical behaviours | Marks |
| * determines the standard deviation * states standard deviation to at least 1 decimal place | 1  1 |

**Question 22 (d) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| The estimated average maximum temperature is  The estimated standard deviation is | |
| Mathematical behaviours | Marks |
| * states average in °*F* * states standard deviation in °*F* | 1  1 |

**Question 22 (e) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| In the model the average value is and the values range from to So and | |
| Mathematical behaviours | Marks |
| * determines *A* * determines *B* | 1  1 |

**Question 22 (f) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| has a minimum when  i.e.  The nearest integer value is So according to  the model, the maximum daily temperatures are  least when i.e. in February. | |
| Mathematical behaviours | Marks |
| * obtains * states the lowest maximum is reached in February | 1  1 |